Excel Sheet Calculates Gas Compressibility Factor

An Excel worksheet calculates the compressibility factor of a real gas for control valve sizing purposes based on tabulated values from the Nelson – Obert Generalized Compressibility Charts.

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The basic gas control valve sizing equation given in ISA-75.01.01-2012 and IEC 60534-2-1 (Second Edition), when arranged to be solved for "C," takes the form shown below:

$$C = \frac{W}{N_6 F_p Y \sqrt{x P_1 \rho_1}}$$

C is defined as being either C_v or K_v depending on the value of N_6. This equation requires a value for the upstream density of the gas where the symbol for density is ρ (rho). The equation assumes that the user knows an accurate value for the density. There are accurate tables for a few gasses, the best example being steam, but such accurate information is not readily available for most industrial gasses. The standards (actually the ISA and IEC standards are, for all practical purposes, identical) make allowance for this fact by including two additional equations. These equations, one based on mass flow units (symbol W) and one on volumetric flow units at standard conditions (symbol Q_s) substitute for the upstream density a calculation of density based on the ideal gas equation using the molecular weight of the gas, its upstream pressure and upstream temperature. To correct for the fact that a calculation of the density of a gas using the ideal gas equation does not always duplicate actual behavior, the standards include the compressibility factor (symbol Z) to include the degree to which a particular gas does not follow ideal gas behavior.

The equation for mass flow is:

$$C = \frac{W}{N_8 F_p P_1 Y} \sqrt{\frac{T_1 Z}{x M}}$$

A method for determining the value to use for the compressibility factor is outside the scope of the standards.

Experience has shown that for most gasses used in industrial processes, and at the pressures and temperatures that they are normally used, for valve sizing purposes, assuming a compressibility factor of 1.0 is usually (but not always) sufficient. There are equations of state that predict the real density of gasses, with varying degrees of

accuracy. Some of these are included with piping system analysis applications. These applications can be expensive, at least from my point of view.

In 1987 when I was developing Hammel-Dahl's first control valve sizing application, I chose to use van der Walls' equation of state, though not as accurate as more complex equations to account for the non-ideal behavior of gases. Advantages of using van der Walls' equation is that it is fairly simple and can be written in terms of the compressibility factor and the reduced pressure and reduced temperature of the gas as follows[1]:

$$Z^3 - Z^2 \left(\frac{P_r}{8T_r} + 1 \right) + Z \left(\frac{27P_r}{64T_r^2} \right) - \frac{27P_r^2}{512T_r^3} = 0$$

It is significant to note that this is of the same form as the generalized compressibility chart, namely, $Z = f(P_r, T_r)$. An iterative solution is required.

In later years, whenever I needed to size a valve for gas service when I suspected that the compressibility might become a factor, I would look up the compressibility factor using the Nelson-Obert Generalized Compressibility Charts[2]. These charts can be found on the Web with a Google search. The site where I found the most easily read ones is at:

http://eon.sdsu.edu/testhome/Test/solve/basics/tables/tablesRG/zNO.html

Some time ago I got tired of reading the Charts, which requires some visual interpolation (and eye strain) and made an Excel sheet where I tabulated a lot of reduced pressure and reduced temperature points from the Nelson-Obert charts and included a two dimensional interpolation scheme that yields a compressibility factor for most gasses that I can find the critical pressure and temperature for. Figure 1 is a screen shot of the sheet showing a calculation for carbon dioxide. The calculation in Figure 1 shows a compressibility factor of 0.7. In this case, assuming a compressibility factor of 1.0 when it is really 0.7 would result in a C_v calculation that would be about 20% high. When the compressibility factor decreases, so does the calculated required valve capacity (C_v or K_v).

The Excel sheet mentioned above is the subject of this article.

As long as I can maintain my website, the worksheet can be downloaded at no charge from:
www.Control-Valve-Application-Tools.com
An alternate site is:
www.industrydocs.org/

The worksheet can also be constructed using the instructions below.

Create an Excel worksheet based on images Screen 1 through Screen 5. In these images, color coding of worksheet entries as follows:

Black cell entries are simply text typed into the cell

Red cell entries are actually formulas that are listed in Table A and must be typed into that cell.

Green cell entries are copies of each red cell entry, but instead of typing in the formula, simply COPY the formula in the red cell and paste it into the three cells to the right of the red cell

There are two tables in the worksheet, Table1 and Table2. Type in the numerical values as shown. Then name the tables by highlighting the data shown inside the red box, and from the Excel Formulas tab select "Define name" and type in the name of the table (TABEL1 or TABLE2) and click OK.

The Nelson-Obert charts are said to have an accuracy within 1 to 2 percent for Z values greater than 0.6 and within 4 to 6 percent for Z values of 0.3 to 0.6. The generalized compressibility factor graphs may be considerably in error for strongly polar gases, with errors as great as 15 to 20 percent.

The quantum gases hydrogen, helium, and neon do not conform to the corresponding-states behavior and the reduced pressure and temperature for those three gases should be redefined in the following manner to improve the accuracy of predicting their compressibility factors when using the generalized graphs:

$$T_r = \frac{T}{T_c + 8} \text{ and } P_r = \frac{P}{P_c + 8}$$

where the temperatures are in kelvins and the pressures are in atmospheres

The worksheet is valid for reduced pressures between 0.0 and 20 and for reduced temperatures between 1.0 and 15. The Nelson-Obert low pressure chart gives some data for reduced temperatures less than 1.0, but there is not enough data to lend it to the present method of tabulating data and interpolating between given data points. I think the reason for this scarcity of data is that the authors of the chart were not able to find much good agreement between various gasses and their compressibility factors in the area between the reduced temperature (T_r) = 1.0 isotherm and the saturated vapor line.

Although the worksheet is configured for conventional US units (Degrees F and pounds per square inch absolute) there is a space to the right of the user interface where the user can easily change the conversion factors for other temperature and pressure units. If you change the conversion factors, you can also edit the unit designations in Column B of the user interface to agree with your revised conversion factors. Keep in mind that all the calculations are carried out in absolute pressure and temperature units.

Below is a brief explanation of how the worksheet works.

The worksheet calculates the compressibility factor (Z) based on a table (TABLE1) of tabulated values of compressibility factors taken from the Nelson-Obert Generalized Compressibility Charts for a range of reduced pressures and reduced temperatures. To account for pressures and temperatures between the tabulated reduced pressures and temperatures the worksheet does three sets of linear interpolations.

At the tabulated value of T_r below the user's given value of T_r the worksheet finds Z at the user's given value of P_r by interpolating between the tabulated value of P_r above the user's given value of P_r and the tabulated value of P_r below the user's given value of P_r.

The worksheet then repeats the above process at the tabulated value of T_r above the user's given value of T_r.

The worksheet then has values of Z at the user's input value of P_r at the tabulated values of T_r below and above the user's input value of T_r.

The final step is to interpolate between these two values of Z to find the value of Z at the user's input values of P_r and T_r. You can follow the above process in Columns S, T, U and V, Rows 6 through 21, where each step is briefly described in Column W.

The worksheet determines the correct column to use in TABLE1 by using a VLOOKUP function in TABLE2.

It then determines the correct row to use in TABLE1 by using a VLOOKUP function in Column 1 of TABLE1.

Using the worksheet

The worksheet has room for up to four sets of process conditions, making it compatible with many of the valve manufacturer's control valve sizing programs which typically are configured to perform simultaneous calculations for up to four sets of process data.

The user interface is in the upper left corner of the worksheet.

You can optionally enter the name of the gas and the tag number of the valve for which you are doing calculations, for your reference, if you are going to either save or print out the worksheet.

Simply enter the critical temperature and critical pressure of the gas, and up to four upstream temperatures and pressures. The fields where the calculated compressibility factor(s) appear will remain blank until all required data has been entered.

It is possible to use other units than PSIA and degrees F for either and/or both upstram and critical prssure and temperature. See the area just to the right of the user interface.

For your reference, below the data and results area, the reduced temperature (T_r) and reduced pressure (P_r) being used in the calculation are displayed along with a note stating whether T_r and P_r are within or outside the limits of the worksheet.

If you have entered temperatures or pressures that result in T_r or P_r that are outside the stated limits, the COMPRESS. FACTOR (Z) field will read #NA and the notes below that will state which parameter is out of limits.

When the process approaches the critical point (T_r and P_r both equal 1.0) the compressibility factor changes very rapidly making it difficult to read the charts accurately or interpolate between points accurately. This is especially notable at the critical temperature (T_r = 1.0) and reduced pressures (P_r) between about 1.0 and 2.0. Also, if you look at the medium pressure Nelson-Obert chart, the T_r isotherm of 1.0 is almost vertical as it approaches the critical pressure (P_r = 1.00). This is because in this area, the compressibility factors of various gasses do not all appear at the same point on the T_r = 1.0 isotherm. When the worksheet returns a compressibility factor (Z) of less than 0.35, the compressibility factor will be shown in red to remind users that their process conditions are close to the critical point.

Gas Compressibility Factor (Z) for Control Valve Sizing Purposes

GAS: Carbon dioxide   TAG: PV-123

		COND 1	COND 2	COND 3	COND 4
CRITICAL TEMP	DEG F	88.00 =========>	=========>	========>	
CRITICAL PRESS	PSIA	1070.00 =========>	=========>	========>	
UPSTREAM TEMP	DEG F	90.00			
UPSTREAM PRESS	PSIA	750.00			
COMPRESS. FACTOR (Z)		0.70			

Reduced Temperature (Tr)   1.004
Tr within limits

Reduced Pressure (Pr)   0.701
Pr within limits

| Data input |
| Final results |

Figure 1. Process data for this application yields a compressibility factor of 0.70

Screen 1

	A	B	C	D	E	F
1	**Gas Compressibility Factor (Z) for Control Valve Sizing Purposes**					
2						
3	**GAS:**	Nitrogen		**TAG:**		
4						
5			COND 1	COND 2	COND 3	COND 4
6	**CRITICAL TEMP**	**DEG F**	-232.52	=========>	=========>	========>
7	**CRITICAL PRESS**	**PSIA**	492.52	=========>	=========>	========>
8						
9	**UPSTREAM TEMP**	**DEG F**	100.00	10.00	10.00	-100.00
10	**UPSTREAM PRESS**	**PSIA**	500.00	500.00	1000.00	2000.00
11						
12	**COMPRESS. FACTOR (Z)**		1.00	0.98	0.96	0.85
13						
14	Reduced Temperature (Tr)		2.464	2.068	2.068	1.583
15			Tr within limits	Tr within limits	Tr within limits	Tr within limits
16	Reduced Pressure (Pr)		1.015	1.015	2.030	4.061
17			Pr within limits	Pr within limits	Pr within limits	Pr within limits
18						Data input
19						Final results

Screen 1. User interface

Screen 2

	K	L	M	N	O	P	Q	R
1		Units used	Conv Factor	Conv Factor	**Input Data Converted to Col L Units**			
2		In	From Col B	From Col B	---			
3		Worksheet	Units to	Units to				
4		Formulas	Col L Units	Col L Units	Cond 1	Cond 2	Cond 3	Cond 4
5			(multiplier)	(adder)				
6	**CRITICAL TEMP**	DEG R	1.0000	459.67	227.15	==>		
7	**CRITICAL PRESS**	PSIA	1.0000	N/A	492.52	==>		
8								
9	**UPSTREAM TEMP**	DEG R	1.0000	459.67	559.67	469.67	469.67	359.67
10	**UPSTREAM PRESS**	PSIA	1.0000	N/A	500.00	500.00	1,000.00	2,000.00
11								
12			For consistency, all process and critical conditions					
13			are converted to PSIA and degrees R.					
14								
15			•To convert Deg C to Deg R, Multiplier is 1.8					
16			and adder is 491.67					
17			•To connvert Deg R to Deg R, Multiplier Is 1.0,					
18			and adder is 0.0.					
19			•To convert Deg K to Deg R Multiplier is 1.8					
20			Adder is 0.0					
21			•To convert Deg F to Deg R Multiplirer is 1.0					
22			Adder is 459.67					
23								
24			•To convert pressure units to PSIA,					
25			multiplier is the conversion factor from					
26			your presure units to PSIA					

Screen 2. Unit conversion inputs.

	S	T	U	V	W	X	Y
1							
2							
3		**COMPRESSIBILITY FACTOR CALCULATION (for T r ≥ 1.0)**					
4	Cond 1	Cond 2	Cond 3	Cond 4			
5							
6	2.464	2.068	2.068	1.583	Reduced Temp, Tr (Limited to a min of 1.0, Max of 15)		
7	1.015	1.015	2.030	4.061	Reduced Press, Pr		
8							
9	2.000	2.000	2.000	1.400	Tr1		
10	2.500	2.500	2.500	1.600	Tr2		
11	13.000	13.000	15.000	17.000	Pr1 Column		
12	14.000	14.000	16.000	18.000	Pr2 Column		
13	1.000	1.000	2.000	4.000	Pr1		
14	1.500	1.500	3.000	5.000	Pr2		
15	0.975	0.975	0.955	0.745	Z(Tr1,Pr1)		
16	0.965	0.965	0.960	0.790	Z(Tr1,Pr2)		
17	0.975	0.975	0.955	0.748	Z(Tr1, Pr)	Interpolate above 2 lines	
18	1.005	1.005	1.010	0.855	Z(Tr2,Pr1)		
19	1.005	1.005	1.015	0.880	Z(Tr2,Pr2)		
20	1.005	1.005	1.010	0.857	Z(Tr2, Pr)	Interpolate above 2 lines	
21	1.003	0.979	0.963	0.848	Z(Tr,Pr)	Interpolate Z(Tr2,Pr) and Z(Tr1, Pr)	

Screen 3. Calculations.

TABLE 1. Tabulated values of compressibility factors (Z) taken from the Nelson-Obert charts

Column Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	Tr1	Tr2	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	3.0	4.0	5.0	6.0	7.0	10.00	20.00	20.01

Pr, Reduced Pressure

Tr \ Pr	0.00	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	3.0	4.0	5.0	6.0	7.0	10.0	20.00	20.01
1.00	OOR	0.9950	0.9650	0.9300	0.8900	0.8500	0.8100	0.7600	0.7000	0.6300	0.5200	0.3000	0.2500	0.3050	0.4400	0.5600	0.6800	0.7900	0.9000	1.2400	2.2800	2.2800
1.05	OOR	0.9960	0.9700	0.9400	0.9050	0.8750	0.8350	0.8000	0.7600	0.7200	0.6700	0.6050	0.3300	0.3300	0.4500	0.5600	0.6800	0.7700	0.8800	1.1850	2.2000	2.2000
1.10	OOR	0.9970	0.9750	0.9500	0.9200	0.8950	0.8640	0.8350	0.8050	0.7750	0.7400	0.7000	0.4600	0.4000	0.4700	0.5750	0.6600	0.7600	0.8600	1.1600	2.1400	2.1400
1.15	OOR	0.9980	0.9770	0.9550	0.9300	0.9100	0.8820	0.8600	0.8300	0.8100	0.7850	0.7560	0.6000	0.5000	0.5050	0.6000	0.6850	0.7900	0.8800	1.1600	2.0900	2.0900
1.20	OOR	0.9980	0.9840	0.9600	0.9400	0.9300	0.9000	0.8800	0.8500	0.8350	0.8150	0.7950	0.6700	0.5850	0.5550	0.6250	0.7100	0.8000	0.8900	1.1600	2.0300	2.0300
1.30	OOR	0.9990	0.9850	0.9700	0.9550	0.9400	0.9250	0.9100	0.8960	0.8800	0.8690	0.8500	0.7600	0.7100	0.6560	0.6600	0.7450	0.8250	0.9050	1.1600	1.9600	1.9600
1.40	OOR	0.9990	0.9880	0.9770	0.9670	0.9580	0.9450	0.9350	0.9220	0.9100	0.9000	0.8860	0.8300	0.7900	0.7340	0.7450	0.7900	0.8550	0.9250	1.1600	1.9000	1.9000
1.60	OOR	0.9990	0.9950	0.9880	0.9800	0.9740	0.9700	0.9600	0.9550	0.9500	0.9400	0.9300	0.9100	0.8850	0.8550	0.8550	0.8800	0.9200	0.9600	1.1600	1.8000	1.8000
2.00	OOR	1.0000	0.9970	0.9950	0.9900	0.9600	0.9650	0.9660	0.9620	0.9600	0.9600	0.9750	0.9650	0.9650	0.9600	0.9600	0.9750	1.0050	1.0400	1.1600	1.6700	1.6700
2.50	OOR	1.0000	1.0000	1.0000	0.9990	0.9960	0.9950	1.0000	0.9920	0.9900	0.9800	0.9750	0.9650	0.9950	0.9900	0.9900	1.0250	1.0700	1.1000	1.1850	1.5500	1.5500
5.00	OOR	1.0000	1.0010	1.0020	1.0030	1.0040	1.0050	1.0070	1.0069	1.0100	1.0100	1.0100	1.0200	1.0280	1.0350	1.0600	1.0800	1.0980	1.1500	1.1700	1.3500	1.3500
15.00	OOR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0010	1.0010	1.0010	1.0020	1.0030	1.0030	1.0060	1.0100	1.0180	1.0240	1.0320	1.1030	1.0500	1.0730	1.1400	1.1400
15.01	OOR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0010	1.0010	1.0000	1.0030	1.0040	1.0060	1.0100	1.0180	1.0240	1.0320	1.1000	1.0500	1.0730	1.1400	1.1400

Tr, Reduced Temperature

"OOR" Stands for "Out Of Range"

Screen 4. Worksheet Table 1.

	AY	AZ	BA	BB	BC	BD
1	**TABLE 2. Reduced Pressure**					
2	Specify the TABLE1 column to lookup Z factor in					
3						
4	**Pr Column ***					
5	0.01	3	0.10			
6	0.10	4	0.20			
7	0.20	5	0.30			
8	0.30	6	0.40			
9	0.40	7	0.50			
10	0.50	8	0.60			
11	0.60	9	0.70			
12	0.70	10	0.80			
13	0.80	11	0.90			
14	0.90	12	1.00			
15	1.00	13	1.50			
16	1.50	14	2.00			
17	2.00	15	3.00			
18	3.00	16	4.00			
19	4.00	17	5.00			
20	5.00	18	6.00			
21	6.00	19	7.00			
22	7.00	20	10.00			
23	10.00	21	20.00			
24	20.00	22	20.01			
25	20.01	23				
26						

Screen 5. Worksheet Table 2.

CELL	FORMULA
	Screen 1, User Interface
C 12	=IF(OR($C$6="",$C$7="",C9="",C10=""),"",S21)
C 14	=IF(OR($C$6="",$C$7="",C9="",C10=""),"",O9/$O$6)
C15	=IF(OR($C$6="",$C$7="",C9="",C10=""),"", IF(OR(C14<1,C14>15),"Tr outside of limits","Tr within limits"))
C 16	=IF(OR($C$6="",$C$7="",C9="",C10=""),"",O10/$O$7)
C 17	=IF(OR($C$6="",$C$7="",C9="",C10=""),"",IF(OR(C16<0,C16>20),"Pr outside of limits","Pr within limits"))
	Screen 2, Unit conversion inputs
O 6	=$C$6*$M$6+$N$6
O 7	=C7*$M7
O 9	=C9*$M9+$N9
O10	=C10*$M10
	Screen 3, Calculations
S 6	=IF(O9/$O$6<1,"OOR",IF(O9/$O$6>15,"OOR",O9/$O$6))
S 7	=IF(O10/$O$7<0.01,0.01,IF(O10/$O$7>20,"OOR",O10/$O$7))
S 9	=VLOOKUP(S6,TABLE1,1)
S 10	=VLOOKUP(S6,TABLE1,2)
S 11	=VLOOKUP(S7,TABLE2,2)
S 12	=S11+1
S 13	=VLOOKUP(S7,TABLE2,1)
S 14	=VLOOKUP(S7,TABLE2,3)
S 15	=VLOOKUP(S9,TABLE1,(S11))
S 16	=VLOOKUP(S9,TABLE1,(S12))
S 17	=S15+((S7-S13)*((S16-S15)/(S14-S13)))
S 18	=VLOOKUP(S10,TABLE1,(S11))
S 19	=VLOOKUP(S10,TABLE1,(S12))
S 20	=S18+((S7-S13)*((S19-S18)/(S14-S13)))
S 21	=S17+((S6-S9)*((S20-S17)/(S10-S9)))

Table A. Formulas to be entered into the Excel sheet

References:

1. Van Wylan, G. J., and Sonntag, R.E. Second corrected printing 1968. *Fundamentals of Classical Thermodynamics,* New York: Wiley, p 358

2. Nelson, L.C. and Obert, E.F. *Chemical Engineering* v. 61, No. 7, pp 203-208 (1954)